# Evaluation of the Arching Effect in Cavity with Various Initial Shapes Based on Principal Stress Vector Field

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**Abstract: The arching effect arises during the formation and development of underground cavities and is closely associated with the final collapse of sinkholes. When the arching effect can no longer sustain the weight of the overburden soil or move upward to the surface, where the conditions prevent its further formation, cavity collapse occurs. Therefore, identifying the top boundary of the arching effect, discerning its shape and size, and tracking its evolution throughout the sinkhole formation and development process are critical and highly anticipated tasks. This paper proposes a method for identifying the position of the arching effect in finite element analysis based on singular points where the magnitudes of principal stresses are equal within the principal stress vector field. Concurrently, the arching effect surface is visualized using the principal stress vector field and applied to several cavities with different initial shapes. The characteristics and differences in the arching effect surfaces are analyzed and compared. Results show that varying initial cavity shapes lead to different positions and forms of the arching effect, providing new insights into the study of arching effects in sinkholes.**

**Keywords:** ***Sinkhole, Arching effect, Principal stress field, Singular points, Vector field, Finite element analysis.***

## Introduction

In recent years, sinkhole-related accidents have frequently occurred due to aging underground infrastructure, such as water pipelines, and inadequate maintenance. Incidents like the 2025 Saitama Sinkhole in Japan and the Seoul Sinkhole in Korea resulted in loss of life and property. To prevent similar accidents and ensure timely remediation after their occurrence, it is urgently necessary to study the formation and development processes of sinkholes up to their collapse and propose corresponding countermeasures. In existing research, several studies have investigated the arching effect on sinkholes (Alonso et al., 2021). Syed Umair Ali Naqvi (2020), in his doctoral dissertation, examined the arching effect using the Discrete Element Method (DEM) alongside laboratory-scale trapdoor model tests, focusing on its role in trapdoor mechanisms. Al-Halbouni et al. (2020) conducted DEM-based simulations of sinkhole development, exploring the relationship between the major principal compressive stress and the arching effect, as well as the role of arching throughout the sinkhole evolution process.

However, current research on the initial formation and development processes of sinkholes remains insufficient, especially regarding the tracking of the arching effect and the determination of its position. Although many researchers have studied the shape and position of the arching effect in underground cavities through various experimental and numerical simulation methods, most studies only provide a general range and approximate shape of the arching effect. They do not offer definitive results on their active position, surface shape, and size.

This paper focuses on singular points where principal stress magnitudes are equal within FEM results, analyzing the physical meaning of these points and their indication in sinkhole problems. Based on this, the upper boundary of the arching effect is defined, which provides a deterministic and physically meaningful location and shape of the arching effect. Compared with traditional methods such as DEM, this method is simpler, computationally less expensive, and the generated arching surface is smoother and possesses clear physical interpretation. In this study, the proposed method is applied to several cavity cases with different initial shapes to explore the relationship between the arching effect shape and the initial cavity geometry. This lays the foundation for further research into the formation and final failure mechanisms of sinkholes.

## Methodology

In this paper, singular points in principal stress vector field are employed to get the shape of the arching effect. Singular points are defined as points where two or more principal stresses are of equal magnitude (Hutchison et al., 1983):

$$\sigma_1 = \sigma_3 \quad or \quad \sigma_1 = \sigma_2 \quad or \quad \sigma_2 = \sigma_3 \qquad (1)$$

Where, $\sigma_1$ is the maximum principal stress; $\sigma_2$ is the intermediate principal stress; $\sigma_3$ is the minimum principal stress. Singular points are generally classified into three types. Within the context of this study, the most observed type is the trisector type, whose principal stress distribution pattern is illustrated in Figure 1.

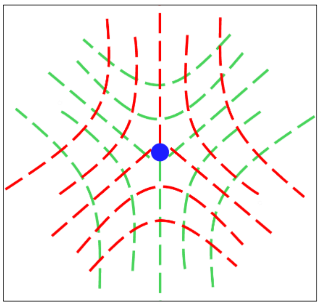

*Figure 1, Trisector-type singular point (blue); red dotted lines represent one principal stress direction, while green dotted lines represent the other.*

Once the positions of singular points are determined, the upper boundary of the arching effect is identified. However, this alone is not sufficient, as many different arching surfaces can pass through the same singular point. Therefore, it is necessary to determine the exact shape and size of the arching surface. In this study, the arching effect surface is generated based on the principal stress vector field. The input is a vector field perpendicular to the direction of the maximum principal stress (compression), which is the direction of the minimum principal stress (tensile). A scalar field is then computed such that its gradient is aligned with this input vector field. The iso-surfaces of this scalar field represent surfaces perpendicular to the input vector field and hence parallel to the maximum principal stress directions. The detailed calculation involves solving (2):

$$\nabla^2 s^k - \nabla \cdot \left( (\boldsymbol{b}^k \cdot \nabla s^k) \boldsymbol{b}^k \right) = 0 \qquad in\ V \qquad (2-1)$$

$$\boldsymbol{n} \cdot (\nabla s^k - (\boldsymbol{b}^k \cdot \nabla s^k) \boldsymbol{b}^k) = 0 \qquad on\ \partial V \qquad (2-2)$$

Where, $k$ is the principal stress direction, $k = 1,2,3$ for 3D analyses, $k = 1,2$ for 2D analyses; $s^k$ is the coordinates to be obtained (scalar field) for $k = 1,2,3$; $\boldsymbol{b}^k$ is the input unit vector field ($|\boldsymbol{b}^k|$=1) for $k = 1,2,3$; $\boldsymbol{n}$ is the outward normal direction vector of the boundaries; $V$ is the problem domain; $\partial V$ is the problem domain boundaries. Example surfaces passing below, though, and above the singular point are shown in Figure 2, The red lines represent the maximum (compressive) principal stress vector field, while the blue lines show three representative contours of the solution to (2).

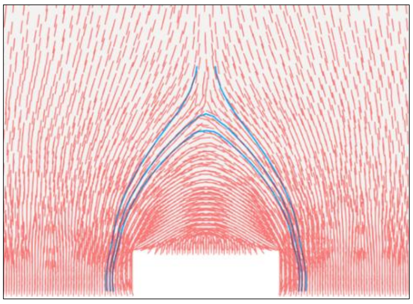

*Figure 2, Schematic illustration of the method for generating arching effect surfaces based on the maximum principal stress vector field.*

The proposed method is then applied to several cavity problems with different initial geometries to investigate the resulting arching effect surfaces, as illustrated in Figure 3. In the figure, the gray regions indicate the soil mass, the white area at the bottom represents the initial cavity, and the curves depict three arching effect surfaces. Among them, the uppermost surface featuring a pointed tip is considered to represent the upper boundary of the arching effect.

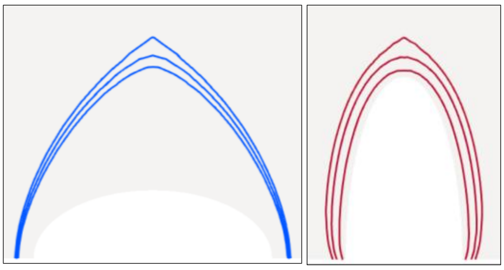

*Figure 3, Two different initial cavity shapes and their corresponding arching effect surfaces.*

## Conclusion

This paper presents a method for determining the location and shape of the arching effect based on the positions of singular points where the magnitudes of principal stresses are equal. This method is grounded in clear physical interpretation, applicable to both 2D and 3D FEM analyses, and yields efficient, consistent results. The analysis results correspond well with existing studies, demonstrating the method's validity. Application to various cavity shapes reveals that the magnitude, height, and upper boundary position of the arching effect vary depending on the cavity's aspect ratio. This indicates a close relationship between cavity shape and arching behavior.

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